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18MAT11

First Semester B.E. Degree Examination, Feb./Mar. 2022
Calculus and Linear Algebra

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Show that the curves $r = ae^{\theta}$ and $re^{\theta} = b$ cut orthogonally. **(06 Marks)**

b. For the curve, $y = \frac{ax}{a+x}$ show that $\left(\frac{2\rho}{a}\right)^{2/3} = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2$ **(06 Marks)**

c. Show evolute of the Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $(xa)^{2/3} + (yb)^{2/3} = (a^2 - b^2)^{2/3}$ **(08 Marks)**

OR

2 a. With usual notations prove that $\tan\phi = r\frac{d\theta}{dr}$ **(06 Marks)**

b. Find the radius of curvature of the curve $r^2 = a^2 \sec 2\theta$. **(06 Marks)**

c. Find the angle between the curves $r = a\log\theta$, $r = \frac{a}{\log\theta}$ **(08 Marks)**

Module-2

3 a. Obtain Maclaurin's series expansion of $\log(1 + \cos x)$ upto the term containing x^4. **(06 Marks)**

b. Evaluate $\underset{x \to 0}{\text{Lt}} \left(\frac{a^x + b^x + c^x + d^x}{4}\right)^{1/x}$ **(07 Marks)**

c. Find the extreme values of the function $f(x, y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$. **(07 Marks)**

OR

4 a. If $u = x^2 + y^2 + z^2$, $x = e^{2t}$, $y = e^{2t}\cos 3t$, $z = e^{2t}\sin 3t$ then find $\frac{du}{dt}$. **(06 Marks)**

b. The temperature T at any point (x, y, z) in space is $T = 400\,xyz^2$. Find the highest temperature at the surface of the unit sphere $x^2 + y^2 + z^2 = 1$. **(07 Marks)**

c. If $u = x^2 - 2y^2$, $v = 2x^2 - y^2$ where $x = r\cos\theta$, $y = r\sin\theta$ then show that $\frac{\partial(u,v)}{\partial(r,\theta)} = 6r^3\sin 2\theta$. **(07 Marks)**

Module-3

5 a. Evaluate $\int_0^a \int_y^a \frac{x}{x^2 + y^2}\,dx\,dy$ by changing the order of integration. **(06 Marks)**

b. Find by double integration, volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ **(07 Marks)**

c. With usual notations, show that the relation between Beta function and Gamma function is $\beta(m, n) = \frac{\gamma(m).\gamma(n)}{\gamma(m+n)}$ **(07 Marks)**

OR

6 a. Evaluate $\int_0^4 \int_0^{2\sqrt{z}} \int_0^{\sqrt{4z-x^2}} dydxdz$ **(06 Marks)**

b. Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dxdy$ by changing into polar coordinates. **(07 Marks)**

c. Prove that $\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta \cdot \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$ **(07 Marks)**

Module-4

7 a. Solve $\frac{dy}{dt} + y\tan x = y^3 \sec x$ **(06 Marks)**

b. Show that the family curves $y^2 = 4a(x + a)$ is self orthogonal. **(07 Marks)**

c. Solve $x^2p^2 + xyp - 6y^2 = 0$ by solving for p. **(07 Marks)**

OR

8 a. Solve $(x^2 + y^3 + 6x)dx + xy^2\, dy = 0$. **(06 Marks)**

b. If the air is maintained at 30°C and the temperature of the body cools from 80°C to 60°C in 12 minutes, find the temperature of the body after 24 minutes. **(07 Marks)**

c. Solve $y^2(y - xp) = x^4p^2$ using substitution $X = 1/x$ and $Y = 1/y$. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix

$$\begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$$ by elementary transformations. **(06 Marks)**

b. Apply Gauss Jordan method to solve the system of equations
$2x + y + z = 10,\ 3x + 2y + 3z = 18,\ x + 4y + 9z = 16$. **(07 Marks)**

c. Find the largest eigen value and the corresponding eigen vector of the matrix

$$A = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}$$ by Rayleigh's power method. Perform four iterations. Take initial vector as $[1\ 0\ 0]^T$. **(07 Marks)**

OR

10 a. Investigate the values of λ and μ so that the equations
$2x + 3y + 5z = 9,\quad 7x + 3y - 2z = 8,\quad 2x + 3y + \lambda z = \mu$ have
(i) a unique solution, (ii) infinitely many solutions (iii) no solution. **(06 Marks)**

b. Use the Gauss-Seidel iterative method to solve the system of equations $5x + 2y + z = 12$, $x + 4y + 2z = 15$, $x + 2y + 5z = 20$. Carryout four iterations, taking the initial approximation to the solution as (1, 0, 3). **(07 Marks)**

c. Diagonalize the matrix $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$. Hence determine A^4. **(07 Marks)**

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